Complexity as a contrast between dynamics and phenomenology

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Abstract

I will present results from a novel analysis of complexity definitions. I will argue that the vast majority of complexity definitions can be interpreted as requiring different combinations of different technical embodiments of five core criteria for complexity: the three dynamical criteria of the existence of many components, determinism and indeterminism; and the two phenomenological criteria of regularity and irregularity. Furthermore, I will show that – while different complexity definitions require different and even exclusive combinations of these criteria – all complexity definitions require contrasting dynamical and phenomenological criteria, i.e. determinism in combination with irregularity or indeterminism in combination with irregularity. Therefore, a contrast between dynamics and phenomenology appears to constitute the conceptual heart of complexity science. I will then propose that the existence of such dyanmics-phenomenology contrasts should be used as a minimal definition of the concept of complexity. Furthermore, I will show that such contrasts constitute a kind of epistemological emergence.

1 Introduction

Since its inception in the late 1980s, complexity science has evolved into a well-established and highly popularized area of science (e.g., for a description of the field's history, Mitchell, 2009). The ascent of the field has been accompanied a number of foundational claims, ranging from a redefinition of the arrow of time (e.g. Davies, 2003) to the often quoted slogan 'more is different' by Anderson (1972). Such foundational claims have also generated much philosophical debate.

Complexity is universally taken to be a property that characterizes a class of dynamical systems. However, the question of how this property should be defined, i.e. which criteria need to be fulfilled for the label 'complex' to be bestowed on a system, has remained unresolved. During the last thirty years, a large number of different complexity definitions have been proposed: Lloyd (2001, p. 7) lists forty-two different definitions of complexity and considers this collection a 'non-exhaustive list'. Responses to this difficulty in finding an unequivocal definition of the core concept of the field have been different among practitioners and philosophers: while complexity scientists have maintained that a single formal definition is unnecessary and that the labelling of systems as being complex can often be underdone intuitively (e.g. Gell-Mann, 1995; Gershenson, 2008), philosophers have been more concerned with identifying which properties are necessary for a system to be called 'complex' (e.g. Zuchowski, 2012; Ladyman et al., 2013). The driving force behind such philosophical accounts is usually not just the development of a better understanding of the concept itself but also its demarcation from related concepts like chaos and randomness. However, even philosophers usually do not aim at deriving a single, authoritative definition of complexity but rather at the identification of sets of criteria that have been associated with the label 'complex' (Ladyman et al., 2013) or at the derivation of minimal definitions, which deliberately highlight the lack of agreed upon criteria (Zuchowski, 2012).

In this paper, I will take a novel approach to the investigation of complexity definitions that – in addition to identify general criteria used in complexity definitions – focuses on the relationship between these criteria and uses these relationships to derive a minimal definition of the concept of complexity. Thereby, I will use a tiered analytical framework (section 1.1, Figure 1) that distinguishes between a general concept, which can be defined through a minimal definition; different definitions associated with this concept; criteria that are used in these definitions; and technical embodiments of these criteria. My results will be illustrated on three well-known models in complexity science (section 2): the CA110 (section 2.1); the Bak-Sneppen model (section 2.2); and the logistic equation (section 2.3).

My analysis can be visualised roughly as an ascent through the different tiers of the analytical framework. Firstly, in section 3, I will argue that the vast majority of complexity definitions can be viewed as requiring different combinations of different technical embodiments of five core criteria for the diagnosis of complexity: the three dynamical criteria of the existence of many components (section 3.1.1), determinism (section 3.1.2) and indeterminism (section 3.1.3); and the two phenomenological criteria of regularity (section 3.2.1) and irregularity (section 3.2.2).

Secondly, in section 4, I will then use my identification of the criteria for the diagnosis of complexity to analyse three different complexity definition, each of which can be seen as indicative of a class of similar definitions. In particular, I will show that the determinism-based definition of complexity by Wolfram (1984, 2002) requires fulfilment of the criteria of determinism, regularity and irregularity (section 4.1); that the indeterminism-based definition by Ladyman et al. (2013) requires fulfilment of the criteria of the existence of many components, indeterminism and regularity (section 4.2); that the inclusive definition by Goldenfeld and Kadanoff (1999) requires fulfilment either of the criteria of determinism and irregularity or of indeterminism and regularity (section 4). My analysis enables a detailed comparison of these definitions and it will become apparent that the determinism- and indeterminism-based definition are exclusive of one another, i.e. there is no overlap between their extensions. In contrast, the extension of the inclusive definition includes the extensions of both other definitions. Furthermore, I will show that the determinism- and indeterminism based definitions both exclude chaotic systems while the inclusive definitions allows these systems to be additionally classified as complex. This will also be borne out by an application of these definitions to the three case studies.

Thirdly, in section 5, I will use the results from my analysis of different complexity definitions to provide a minimal definition of complexity, i.e. to provide a description of the concept of complexity that underlies all of these definitions. The minimal definition will be based on a property shared by all analysed definitions: they all require combinations of contrasting dynamical and phenomenological criteria. In particular, all definitions require either the dynamical criterion of determinism in conjunction with the phenomenological criterion of irregularity or the dynamical criterion of indeterminism in conjunction with the phenomenological criterion of regularity. Furthermore, I will show that two of the most prevalent metaphorical descriptors of complexity, 'being between order and chaos' and 'self-organisation' can also be interpreted as encapsulations of specific dynamics-phenomenology contrasts, namely the one specific to the determinism- and the indeterminism-based definition, respectively. Accordingly, I will propose that the concept of complexity should be (minimally) defined as the existence of dynamics-phenomenology contrasts. Additionally, I will show that the dynamics-phenomenology contrast that is reflected in the definitions and descriptors can be viewed as a specific kind of epistemological emergence (section 5.2).

The realisation that it is this contrast between (deterministic/indeterministic) dynamics and (regular/irregular) phenomenologies that is articulated in all complexity definitions and the major metaphorical descriptors of the field, and therefore forms the conceptual heart of complexity science, constitutes the main result of my analysis. In light of this result, the coexistence of many different complexity definitions can be viewed as providing a means of identifying this core concept in different classes of systems. While the relative merits of different definitions can still be argued, their coexistence should therefore not be seen as a sign of deep conceptual divisions but as a means of highlighting one shared concept, i.e. the existence of a contrast between dynamics and phenomenology, in many different models. Accordingly, my analysis also offers a way to demarcate the field of complexity science itself: namely, as the field of science concerned with the study of systems that display such contrasts between their dynamics and phenomenologies. Since the concept of complexity can be interpreted as a kind of epistemological emergence, this also implies that emergence is indeed part of the foundations of complexity science.

In addition to those conceptual results about the foundations of complexity science, my analysis also leads to the clarification of a number of concepts, definitions and metaphors in complexity science. It therefore results in a clear exhibition of the epistemic structure

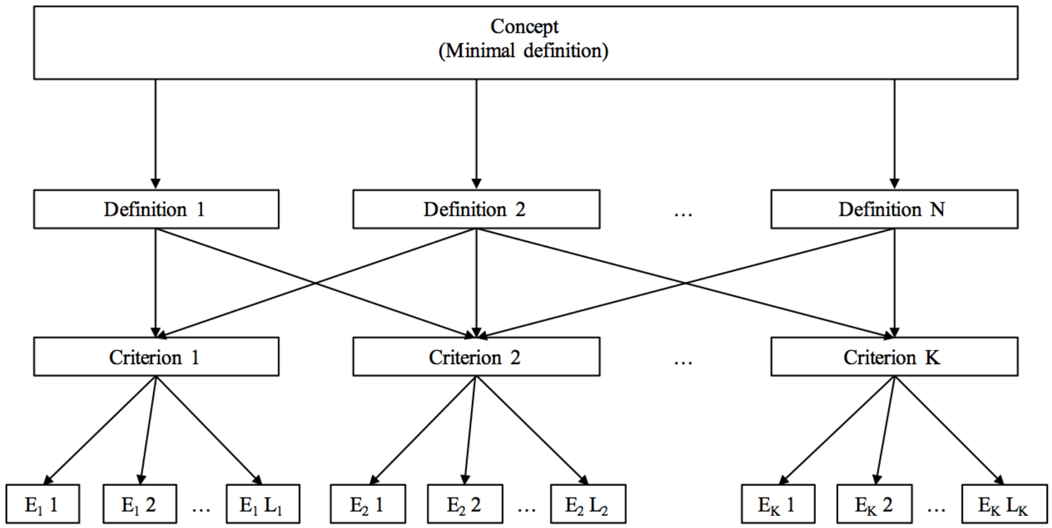

Figure 1: Relationships between concepts, definitions, criteria and embodiments (abbreviated as E). The total number of definitions, criteria and embodiments are denoted by N, K and L, respectively.

of the field, i.e. it allows for a conceptual sharpening of terminology and reveals the relationships of different terms with each other. Accordingly, I hope that my work here also contributes to the terminological tidying of complexity science that has been requested by several authors (e.g. Horgan, 1995; Frigg, 2003). However, since this analysis is an exercise in rational reconstruction, I do not claim that it is the only viable interpretation – nor that it always captures the initially intended meaning of a concept in all historically relevant nuances – or that the analytic framework I use is the only possible one for the analysis of scientific definitions. Instead, I hope that merits of my conceptual reconstructions will be evident in the clarity with which they expose the concept underlying different complexity definitions and the relationships these definitions have with each other.

In the following two subsections, I will briefly review the frameworks and concepts on which my analysis in sections 3 – 5 will be based: the tiered framework for the analysis of definitions (section 1.1); and the concept of emergence, which – relative to the wealth of material available on this topic – will only be reviewed very briefly (section 1.2).

1.1 Concepts, Definitions, Criteria and Embodiments

The coexistence of many different definitions of a core concept is not unique to complexity science: similar constellations can be found for a large variety of terms, e.g. 'partnering' (Nystroem, 2005); 'sensitivity' (Mencattini and Mari, 2015); 'synergy' (Berthoud, 2013) and 'chaos' (Zuchowski, 2017). The conceptual frameworks used to analyse the relationships between the overarching concept and the different definitions are usually based on a further decomposition of the latter into different 'components' (e.g. Nystroem, 2005) or 'criteria' (e.g Zuchowski, 2017). I will adopt the latter nomenclature. In a third level of analysis, the criteria required by a given definition can be further decomposed into technical embodiments, i.e. formal specifications of a given criterion that allow a quantitative measurement in a given set of scenarios (Mencattini and Mari, 2015; Zuchowski, 2017, e.g.). From such a compositional analysis of a concept into separate definitions, criteria and embodiments, conclusions about this concept can be drawn. For example, Nystroem (2005) concludes that all definitions of 'partnering' require the criteria of 'trust' and 'mutual understanding', thereby rendering these two properties the minimal conditions for a scenario to be given this label.

In this paper, I will adopt a similar four-tier framework for the compositional analysis of the concept of 'complexity'. The four tiers of the framework - general concept, definitions, criteria and embodiments - are displayed in Figure 1. This compositional analysis will allow me to identify a feature characteristic of all of these definitions: despite the fact that the combinations of criteria used in these definitions may vary, all definitions implicitly postulate a contrast between the dynamical and phenomenological criteria required (section 5). I will the propose that this contrast should be seen as characteristic of the concept of complexity and could be used for a minimal definition of this concept. In the following, I will briefly describe each component of the framework; however, none of the terms is used with radically different meaning than that colloquially assigned to it.

Concepts The term 'concept' is used to denote the overarching notion that underlies all (or most) of the definitions in question. It is not guaranteed that each compositional analysis in the style of the one displayed in Figure 1 will reveal an underlying concept: for example, Berthoud (2013) comes to the conclusion that the disparaging definitions associated with the term 'synergy' (in medicine) are to dissimilar to clearly identify a concept behind them and advocates that the meaning of the label should be fixed by one statistical definition. However, in other cases, e.g. for the terms 'partnering' (Nystroem, 2005) or 'chaos' (Zuchowski, 2017), the analysis of a set of definitions associated with these terms reveals enough similarities that the definition of an overall concept is warranted. There is an obvious linguistic pitfall here in the use of the terms 'define' and 'definition': while it is used in this framework to denote more specific descriptions of the general concept, it is also possible to 'define' this very concept, i.e. to provide a clear statement of the notion associated of a label like 'chaos' or 'complexity'. In order to avoid confusion, I will refer to such a statement as the minimal definition of a concept, i.e. a definition that is in some way reflected by all separated definitions associated with the concept.

Definitions In this framework, the term 'definition' is used in its most common meaning, i.e. to denote a statement that specifies a set of criteria, which need to be fulfilled by a given system to receive the concept label in question.

Criteria The term 'criteria' is used to denote properties that a system needs to have in order for this system to be given a particular concept label. They are therefore necessary conditions for the diagnosis of the conjunctive property corresponding to the label. This implies that definitions are viewed as bi-conditional relations: i.e. if the required properties are present, then the system will receive the label specified by the definition and if a system carries a certain label, then all properties specified in this definition will be fulfilled. Criteria are therefore necessary conditions for the diagnosis of the concept property as defined in this specific definition.

The medically inspired terms of 'criteria' and 'diagnosis' rather than 'conditions' have been deliberately chosen: it facilitates the drawing of a clear distinction between the classification of a system and the identification of conditions under which the system will likely be classified as such, i.e. between the definition of concept as a set of required properties and the necessary and sufficient conditions for a display of these properties. This does not mean that these two sets of conditions cannot overlap, i.e. that necessary conditions used as criteria cannot also be viewed as necessary or sufficient conditions for the occurrence of the labelled behaviour. However, Suárez (2013) and Zuchowski (2017) argue (convincingly, in my opinion) that scientists' discourse shows a general distinction between necessary conditions used in definitions (i.e. criteria) and sufficient conditions for the occurrence of the behaviour thus defined. It should also be noted that Ladyman et al. (2013) appear to pursue a similar strategy and only include those conditions they consider necessary for complexity into their eventual complexity definition (section 4.2).

It will be useful for my analysis to adopt another distinction introduced by Zuchowski (2017, Chapter 3): that between 'phenomenological' and 'dynamical' criteria. The designation 'dynamical' thereby refers to properties of the underlying mechanisms of a system, e.g. the formalism of a model, while the designator 'phenomenological' refers to those properties of a system's behaviour that are observable without any knowledge about these underlying mechanisms, e.g. the output of a model. At first glance, one might assume that dynamical properties should always be viewed as sufficient or necessary conditions for the occurrence of a certain behaviour rather than as criteria for its diagnosis. However, However, Zuchowski (2017, Chapter 3) shows that a closer analysis of existing chaos definitions reveals that the label 'chaotic' will only be given to a system if the dynamical criterion of determinism is also fulfilled. In some chaos definitions, this particular criterion is crucial to distinguishing the concept from other, related concepts: in particular, in the definition of stochastic chaos. The definition of stochastic chaos requires phenomenological indistinguishability from systems whose underlying mechanisms are Bernoulli processes, i.e. from truly stochastic systems. In this definition, the dynamical property of determinism is therefore used as a criterion for the diagnosis of chaos rather than postulated as a sufficient or necessary condition for its occurrence. In section 3, I will show that there are several dynamical criteria for complexity and that these play a similar demarcating role as the one identified by (Zuchowski, 2017, Chapter 3).

Embodiments Since the quantitative measurement of complexity is not one of my main concerns in this paper, the fourth step in the compositional analysis, i.e. the analysis into different 'embodiments' will not be pursued in detail. The term 'embodiment' thereby denotes a statement that specifies a criterion in technical detail, usually so that a quantitative measurement of the property can be performed in a given class of scenarios. While the specification of different embodiments for a given criterion will not be a main focus point of this paper, Zuchowski (2017) and Mencattini and Mari (2015) have found that the existence of different embodiments of a criterion adds further variability to the number of definition for a given concept: in addition of requiring different combinations of criteria, definitions of a concept can also differ by requiring the same combination of criteria but in different technical embodiments.

1.2 A (very brief) note on emergence

The concept of emergence was initially developed within the tradition of the logical analysis of scientific theories. Accordingly, it was coupled with the notion that scientific theories and explanations can be analysed in formal, logical and linguistic terms. In particular, Hempel and Oppenheim (1948, p. 138) describe explanation as the logical deduction of the description of an empirical phenomenon (the explanandum) from antecedent conditions and general laws (the explanans). Emergence is then a conceptualisation of the failure to derive an explanandum from the salient explanans, typically describing the spatial, even microscopic, constituents of the system (Hempel and Oppenheim, 1948, p.147):

> Generally speaking, the concept of emergence has been used to characterise certain phenomena as "novel", and this not merely in the psychological sense of being unexpected, but in the theoretical sense of being unexplainable, or unpredictable, on the basis of information concerning the spatial parts or other constituents of the system in which the phenomena occur [...].

Much of the more recent discussion of emergence has taken place outside the formalised, unified framework for science aspired to in these early analyses (e.g., for review, Bedau and Humphreys, 2008). Instead of being derived from a logical analysis of theories, recent notions of emergence have mostly been constructed from catalogues of defining instances. Hempel's and Oppenheim's (1948) original desire to raise the notion beyond that of the psychologically puzzling is thereby not always upheld: Ronald and Sipper (2001, p. 20) propose that a feeling of surprise should be the central element of an "emergence test". Butterfield (2011, p. 922) pointed out that, unsurprisingly, notions of emergence constructed from different catalogues of defining instances will be widely different.

However, for the purpose of this study, a particular reading of the definition as devised by Hempel and Oppenheim (1948, p.147) will suffice: I will use the term emergence to describe the existence of features in the phenomenology of a system, which could not have straightforwardly been predicted from knowledge about the essential properties of the system's dynamics. While this rephrasing of the definition emphasises the conceptual division between the dynamics and the phenomenology of a system and thereby renders the concept easily combinable with the framework for the analysis of complexity definitions introduced in section 1.1, it is also general enough to be compatible with the majority of conceptualisations of emergence in complex system (e.g. Gregersen, 2003; Bedau and Humphreys, 2008).

The notion of emergence described here is clearly an epistemological one (e.g. Silberstein and McGeever, 1999; Cunningham, 2001): it is fundamentally an assessment of the predictiveness of knowledge about a certain part of a system (i.e. its dynamics) with respect to another part of the system (i.e. its phenomenology). While the distinction between epistemological and ontological emergence has been notoriously difficult to define (e.g. Silberstein and McGeever, 1999; Butterfield, 2011), I maintain that the type of emergence seen in the systems usually discussed in complexity science (e.g. the three case studies introduced in section 2) cannot be ontological in the sense that is hypothesised to underlie emergence in systems that are usually cited as examples for the latter kind of emergence. In particular, the kind of emergence seen in complex systems does not appear to involve the hypothesised changes in the dynamics of the system that have been labelled as ontologically emergent (i.e.

the change from non-probabilistic to probabilistic dynamics at the quantum boundary). In contrast, it is obvious in the systems discussed in section 2 that the any emergent features - however novel and useful they are - can in principle be reduced to a definite set of dynamics.

The notion of epistemological emergence inherently contains an element of observer-dependency: any assessment of predictiveness will always be relative to the predictive power of the individual performing the prediction. However, this observer-dependency is usually treated as philosophically unproblematic: the hypothetical observer is assumed to be the 'best possible' one, e.g. in most scenarios, a competent expert who can reasonably be expected to possess maximum predictive powers with respect to the given information. Similarly, I will take the notion of 'prediction' in the definition of emergence tendered here to mean 'prediction' by a competent, human expert.

However, even with this grounding of prediction, the notion of emergence I work with in this paper – and that I will eventually ascribe to complex systems as they are prevalently defined (section 5) – is clearly a 'weak' one. In fact, it is even less demanding then the notion of emergence for which Bedau (2008) has coined the label 'weak' since his notion additionally requires the possibility of ascribing causal powers to the emergent features. However, this does not necessarily render this notion of emergence an uninteresting one. Firstly, in section 5.2, I will argue that it correctly captures a true physical feature of the class of systems usually described as complex; if this feature is less 'spectacular' than anticipated, then this is valuable information in itself. Secondly, the fact that there are systems whose behaviour is not straightforwardly predictable from its dynamics is not without conceptual interest and its realisation could inspire new investigative approaches – as requested by a number of complexity scientists (e.g. Wolfram, 2002; Kauffman, 2008; Mitchell, 2009).

2 Case studies: complex and chaotic systems

In this section, I will briefly describe three seminal models that will be used to illustrate both the criteria required by different complexity definitions (section 3) as well as the differences in their extensions (section 4).

The three models discussed here are both relatively simple as well as generally well known. Accordingly, my own descriptions of their dynamics and their phenomenologies will be kept as brief as possible. The reliance on such simple models in the discussion of complexity definitions has been criticized, e.g. Morowitz (2002) and Fromm (2004) argue that 'true' complexity cannot be found in such simple models and that the focus of the discussion should shift to more realistic representations of the 'complicated' systems found in nature. However, the vast majority of the practitioners (e.g. Holland, 1998; Bak and Sneppen, 1993) and philosophers (e.g. Bedau and Humphreys, 2008; Hooker, 2011) discussing complexity definitions still rely heavily on these models and this is certainly the case for the body of literature I am directly engaging with in this paper (Goldenfeld and Kadanoff, 1999; Wolfram, 2002; Ladyman et al., 2013). Accordingly, the choice of these models as illustrations of the criteria used in these definitions and of the concept that I claim underlies all of these definitions is justified by the immediate methodological context of the discussion. This does not preclude an extension of this discussion to more 'complicated', realistic systems in the future, of course.

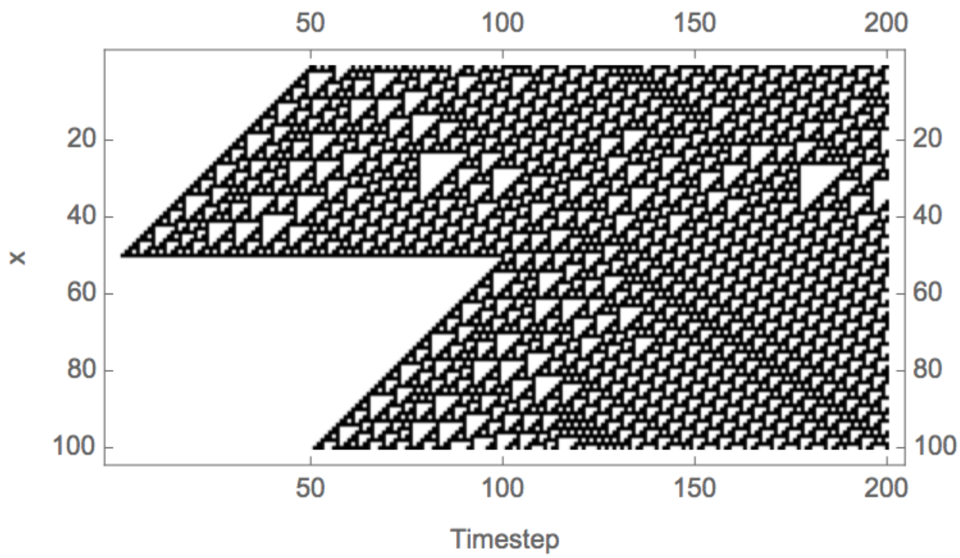

Figure 2: Phenomenology of the deterministic cellular automaton with two-valued, nearest-neighbours rule set 110. The model was initiated with a single black cell in the middle of the domain and has periodic boundary conditions.

The three models described here are: the CA110, which is a deterministic cellular automaton (section 2.1); the Bak-Sneppen model, which is an indeterministic cellular automaton (section 2.2); and the logistic equation, which is a chaotic mapping (section 2.3). I have chosen these particular models as case studies as each one has been used as an exemplar for a different definition of complexity: they can therefore be seen as providing the clearest examples for the combination of criteria required by each of the definitions discussed in section 4. Accordingly, they also serve to highlight the contrasts between the extensions of different complexity definitions.

2.1 Deterministic cellular automaton: CA110

The CA110 is a one-dimensional, nearest-neighbours cellular automaton, i.e. it consists of a grid (strip) of N cells, which can assume one of two colour states, 'black' and 'white'. The colour of a cell at position i is determined by the colours of itself and its two neighbouring cells at position $i-1$ and $i+1$. For two colour states, there are eight different combinations that these three cells can assume. The rule-set of the CA110 therefore comprises eight rules of how the cell i is to respond to each of these combinations: e.g. if both cells $i-1$, i and $i-1$ are black at time step t, then cell i will become white at time step $t+1$; if cell $i-1$ is white and cells i and $i+1$ are black at time step t, then cell i will remain black at time step $t+1$; and so on. The CA110 has been studied extensively by several authors; a complete description of its rule set can be found in e.g. Wolfram (2002). For the purposes of this paper, it is only important to note that the dynamics of the CA110 are fully deterministic: provided the specification of appropriate boundary conditions, the states of all N cells at time step $t+1$ is uniquely determined by the states of these cells at time step t.

Figure 2 shows the phenomenology of a CA110 with $N = 100$, periodic boundary conditions and an initial state of just one black cell at $i = 50$. It is immediately obvious that the phenomenology of the CA110 – if represented on a two-dimensional space-time

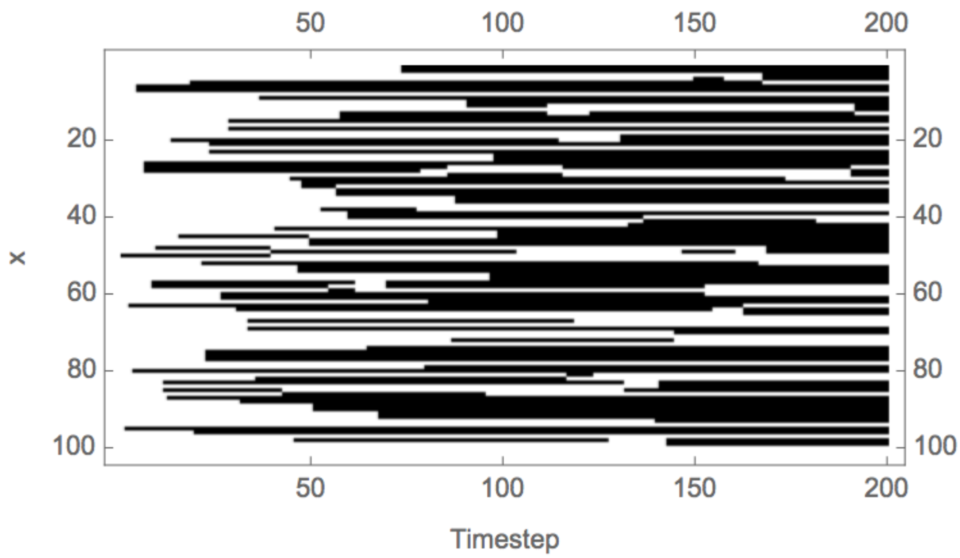

Figure 3: Phenomenology of the indeterministic Bak-Sneppen model. The model was initiated with a single black cell in the middle of the domain and has periodic boundary conditions.

plot (i.e. a representation in which color changes on the strip are shown as two-dimensional stationary patterns) – is dominated by local pattering of triangular areas of different sizes. It is notable that areas in the space-time diagram dominated by collections of triangles of a given size roughly indicate areas on the ring which change colour with a given periodicity. Accordingly, the space-time diagram could also be read as an indication that the behaviour of the CA110 is dominated by both areas that change colour without a determinable frequency, i.e. flicker randomly, and areas in which a dominated frequency can be identified.

2.2 Indeterministic cellular automaton: Bak-Sneppen model

The set-up for the (discrete) Bak-Sneppen model is very similar to that of the CA110: a one-dimensional, nearest-neighbours cellular automaton with two colour states. However, in contrast to the CA110, the dynamics of the Bak-Sneppen model are probabilistic. During each time-step t, a cell i is chosen randomly. If this cell i is currently white, then its colour value and that of its two nearest-neighbours at sites $i-1$ and $i+1$ will be replaced by randomly chosen colour values, i.e. there is an equal probability of $p = 0.5$ that each of these cells will be black or white at time step $t+1$. If the cell i is black, then no changes are made to the automaton. It is immediately obvious that the dynamics of the Bak-Sneppen model are not deterministic: the state of the automaton at time t does not uniquely determine the state of the automaton at $t+1$.

Figure 3 shows the phenomenology of a Bak-Sneppen model with $N = 100$, periodic boundary conditions and an initial state of just one black cell at $i = 50$. While the original version of the model uses a continuous colour spectrum (Bak and Sneppen, 1993), it has been determined that the phenomenology of the discrete model is similar to that of the continuous one in all essential aspects (Meester and Znamenski, 2002). As apparent in Figure 3, the behaviour of the model is characterised by periods of quiescent behaviour, which is punctuated by shorter and longer periods of rapid, localised change. For longer

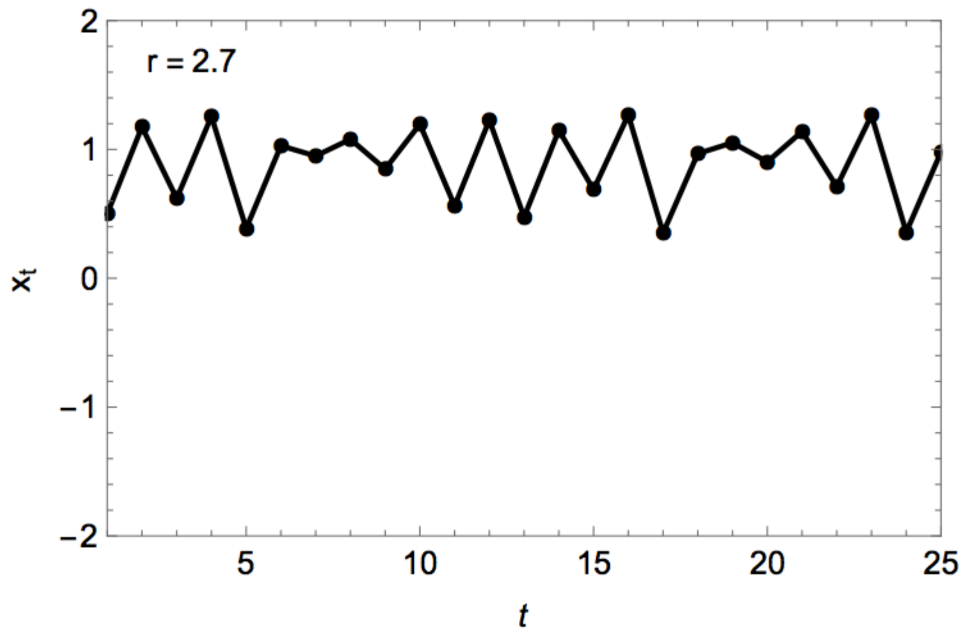

Figure 4: Phenomenology of the logistic equation. The model was initialized with $x_0 = 0.5$.

runs of the model, it can be shown that, under many conditions, the spatial size of these 'catastrophic events' or 'avalanches' and the frequency with which thus events appears are approximately related by a power-law (e.g. Bak and Sneppen, 1993; Meester and Znamenski, 2002), i.e. small events of rapid change occur exponentially more often than large events of rapid change.

2.3 Chaotic mapping: the logistic equation

The logistic equation is a well-known discrete mapping, which has served as a paradigm case for certain definitions of chaos (e.g. May, 1974; Smith, 1998; Zuchowski, 2017). The mapping is non-linear and consists of the following equation:

$$x_{t+1} = x_t \left(1 + r \left(1 - x_t\right)\right), \tag{1}$$

where r is a fixed parameter that is usually called the growth rate. The function is fully deterministic: given a value x_t, it uniquely specifies a value for x_{t+1}. However, in contrast to the cellular automata discussed in sections 2.1 and 2.2, the logistic function consists of a sequence of single-points and its space-time diagram is therefore a one-dimensional sequence of points.

The behaviour of the mapping varies for different values of r. In the chaotic regime, which happens for values of $2.57 < r < 3$, the function shows an aperiodic phenomenology, i.e. the sequence of points ... x_{t-1}, x_t, x_{t+1} ... appears to be random (Figure 4). Furthermore, in this regime, the function displays sensitive dependence to initial conditions (SDIC): small changes in the initial (or any intermediate value) of x usually lead to large difference in the further development of the sequence. These properties of the logistic equation, which lead to a phenomenology that is in significant aspects indistinguishable from the phenomenology of a truly random, i.e. probabilistic, process, have been traced back to the fact that the logistic equation can be expressed in terms of the shift function (e.g. Tsonis, 1992).

3 Dynamical and phenomenological criteria for complexity

In this section, I will identify and discuss the dynamical and phenomenological criteria (section 1.1) used in the majority of complexity definitions. Several authors (e.g. Ladyman et al., 2013; Hooker, 2011) have provided lists of conditions for complexity. These lists are often relatively long and do not always distinguish between (i) criteria for the diagnosis of complexity and conditions for its occurrence; and (ii) concepts, definitions, criteria and technical embodiments (as introduced in section 1.1). I will therefore build upon these existing works by showing that these longer lists of conditions can be distilled into five criteria for complexity (some of which have different technical embodiments).

Three of these criteria are dynamical criteria (section 3.1): the existence of many components (section 3.1.1); determinism (section 3.1.2); and indeterminism (section 3.1.3). In addition, there are two phenomenological criteria (section 3.2): regularity (3.2.2) and irregularity (section 3.2.1). The most prevalent embodiments of each criterion will be discussed in the relevant sections.

3.1 Dynamical criteria

In this section, I will discuss the dynamical criteria used in complexity definitions. As defined in section 1.1, dynamical criteria are criteria that refer to properties of the underlying mechanisms of a system, e.g. to the formalism of a model. Deciding whether a given system fulfils these criteria therefore requires knowledge about the dynamics of the system. This does not exclude the possibility of inferring these dynamical properties from the phenomenology of a system: in fact, for natural systems, this is often necessary and might require additional diagnostic techniques (e.g., on the use of such techniques in chaos theory, Zuchowski, 2017, Chapter 4). In contrast, the equations of models are usually known and their fulfilment of any dynamical criteria can then be directly determined.

3.1.1 The existence of many components

Many complexity definitions require the existence of many components as a dynamical criterion for complexity (Zuchowski, 2012, p. 212; Ladyman et al., 2013, p. 35; p. 57). As Zuchowski (2012, p. 212) points out, the criterion of the existence of many components is sufficient to distinguish complex systems from virtually all chaotic systems: chaos definitions are virtually exclusively defined for maps, i.e. for systems with few degrees of freedom (e.g. Devaney, 1989; Hilborn, 2002). Accordingly, definitions that do not require the existence of many components as a criterion for the diagnosis of complexity are usually associated with relatively expansive accounts of the scope of complexity science and include chaos theory as one of its subfields (e.g. Casti, 1992a,b; Hooker, 2011).

The demarcating power of this criterion is also obvious when applied to our set of case studies: both the Bak-Sneppen model (section 2.2) and the CA110 (section 2.1) are many-component models and hence fulfil this criterion; in contrast, the logistic equation (section 2.3) is a one-dimensional mapping and therefore does not.

3.1.2 Determinism

Determinism (as a property label) usually refers to the fact that the equations comprising the canonical description of the dynamics of a given system admit a unique development for each initial (or intermittent) value and therefore do not contain any probabilistic terms.

The use of the concept of determinism in complexity science is characterised by an idiosyncratic feature: it is usually the second aspect of my definition above, i.e. the absence of any probabilistic terms, that is stressed. This emphasis is evident in the labelling of a prominent class of models capable of being complex: cellular automata (CAs), which are one of the paradigmatic classes of models in complexity science, are usually categorised as being 'probabilistic' or 'non-probabilistic' (e.g. Wolfram, 2002; Batty, 2005). Non-probabilistic CAs – like the CA110 (section 2.1) – do not have any probabilistic terms in their rule sets; while probabilistic CAs – like the Bak-Sneppen model (section 2.2) have rule sets with probabilistic terms. An identical categorisation is used for two other prevalent classes of models in complexity science: network and agent-based models. A consequence of this focus on the presence or absence of probabilistic dynamics is the fact that, if a system's underlying processes or formalisms are known, then one can straightforwardly determine whether it is deterministic or not.

3.1.3 Indeterminism

Indeterminism as a dynamical criterion can be defined as a complement to the criterion of determinism: i.e. as the fact that the equations of the canonical description of the dynamics of a system admit more than one development for a given initial (intermittent) value.

Phrased in terms of the presence of probabilistic and non-probabilistic terms, the criterion of indeterminism requires that at least some probabilistic terms are present in the system's dynamics. The criterion of determinism (section 3.1.2) and that of indeterminism are therefore mutually exclusive of each other, i.e. no system can fulfil both criteria simultaneously. In our set of case studies, the Bak-Sneppen model is indeterministic (section 2.2), while the CA110 (section 2.1) and the logistic equation (section 2.3) are deterministic.

Several of the complexity definitions listed by Ladyman et al. (2013, p. 36) require indeterminism as a criterion for the diagnosis of complexity. This is sometimes phrased as a requirement of 'multiple causal pathways' or 'multiple conditional pathways', which appears to be a rephrasing of the admittance of multiple solutions for a given initial value, i.e. of the defining feature of indeterminism. Indeterminism as a criterion for complexity is not identical with a requirement of stochasticity, i.e. the requirement that the dynamics of a system can be represented as a Bernoulli process. Rather, even authors using the descriptor 'disordered' to describe the dynamics of complex systems (e.g. Ladyman et al., 2013, p. 41), effectively only require the presence any probabilistic terms (independent of their underlying algebra and distribution) for a system's dynamics to be counted as such.

While therefore none of the prevalent complexity definitions seems to require stochasticity, the requirement of even some indeterministic terms is sufficient, of course, to ensure that the dynamics of such systems do not comply with the definition of determinism. Accordingly, requiring indeterminism as a dynamical criterion for complexity implies that many models that have traditionally been viewed as complex can no longer be given that label: namely, deterministic CAs (including the CA110 discussed in section 2.1) and deterministic networks (which constitute the foci of many seminal works in complexity science, e.g. Wolfram, 2002).

3.2 Phenomenological criteria

Distinguishing between phenomenological and dynamical criteria in complexity definitions is particularly difficult since these definitions often contain terms that appear to be describing dynamical features of a system (e.g. 'memory', 'hierarchy' etc) but are actually used to denote phenomenological properties. e recall: phenomenological features and criteria are properties of a system's behaviour; they can be identified without knowledge of a natural system's underlying processes or of a model's equations. The use of such pseudo-dynamical terminology appears to be a consequence of a particular approach to the analysis of these models that is based on the proposition of hypothetical 'alternative dynamics', i.e. dynamics, which are known not to be the real processes governing a system, but that would be able to reproduce certain aspects of this system's behaviour.

It is notable that this approach is not based on the claim that the alternative dynamics are equally likely candidates for the 'true' dynamics of the system. Rather it assumes that actual dynamics are known and treats the fact that the phenomenology could (hypothetically) be caused by the chosen alternative dynamics as a phenomenological property. This is very apparent in discussions of the logistic equation (section 2.3): its phenomenology (Figure 4) is often described as being indistinguishable from that of a Bernoulli process (e.g. May, 1974; Lorenz, 1993). However, both from its grounding in population dynamics (May, 1974) and its actual formalism (1), it is apparent that the model is deterministic and therefore fundamentally different from a Bernoulli process. However, the fact that this difference cannot trivially be derived from its phenomenology is seen as an important feature: Lorenz (1993) even proposes to use indistinguishability from a probabilistic process as definition of chaos.

I maintain that it is important to distinguish between phenomenology and dynamics and to therefore look beyond any possibly misleading terminology for two reasons: firstly, in sections 3.2.1 and section 3.2.2, it will become apparent that the large number of different concepts used as phenomenological criteria for complexity can actually be seen as different embodiments of just two such criteria, regularity and irregularity; secondly, a clear categorisation of criteria as dynamical or phenomenological will reveal that most complexity definitions are based on the requirement of the existence of a contrast between the dynamics and the phenomenology of a system. In section 5, I will argue that this fact can be used to derive a minimal definition of the concept (section 1.1) of complexity.

3.2.1 Regularity

I will subsume under the criterion of regularity all requirements that the behaviour of a system shows some form of identifiable patterning. In keeping with the embodiments of this criterion required by complexity definitions, I therefore define regularity (as a property label) more loosely than might be done in other contexts and will not require that specific frequencies, i.e. spatial or temporal intervals for the repetition of a phenomenological feature, must be identified for a system to be (phenomenologically) regular. However, for some embodiments of the criterion of regularity in complexity science, even such a strong version of the criterion would be fulfilled.

In complexity definitions, regularity is usually required to exists in some parts of system's phenomenology only, i.e. most complexity definitions do not require that a system displays fully periodic or regular behaviour. As a matter of fact, many complexity definitions explicitly require that the phenomenology of the system is regular in some parts and irregular in others (Ladyman et al., 2013, p. 12; section 4.1). Therefore, the phenomenological criteria of regularity and irregularity (section 3.2.2) are not exclusive of one another.

Embodiments of regularity The most frequently used embodiment of the criterion of regularity is the requirement that distinct ***patterns*** can be identified in the system's phenomenology (e.g. Ladyman et al., 2013, p. 12). The term thereby refers to a visually identifiable pattern in some representation of the system's behaviour: in system's that have multiple spatial dimensions, such patterns are usually distinct static or dynamical spatial structures (e.g. Gliders in the Game of Life); in one-dimensional system's such patterns are usually identified as static structures in spatial-temporal plots (e.g. the Sierpinski triangle in the CA30. In the CA110 (section 2.1, Figure 2) one can identify local areas in which triangular structures appear to be regularly stacked upon each other. What counts as a pattern is not always explicitly defined; however, it is usually assumed that such structures can reliably be identified through visual inspection (e.g. Wolfram, 2002).

A numerical embodiment of the criterion of regularity is that of ***pattern entropy***. Pattern entropy measures count the number of patterns in a suitable representation of a model's phenomenology. Examples of pattern entropies are block entropy (Wolfram, 2002) and statistical complexity (Feldman and Crutchfield, 1998). Pattern entropies are highest for systems whose behaviours that show many different periodicities, e.g. for superpositions of oscillations with different frequencies or amplitudes. These numerical embodiments of regularity are therefore usually more demanding than a simple identification of patterns.

A further embodiment of the criterion of regularity is the requirement of the existence of ***macrolaws***. Holland (1998, p. 27) describes this embodiment in the following way:

> Persistent patterns often satisfy macrolaws. When a macrolaw can be formulated, the behaviour of the whole pattern can be described without recourse to the microlaws [...]. Macrolaws are typically simple relative to the behavioural details of the component elements. The law describing the behaviour of the glider in in Conway's model universe is a clear example.

Macrolaws are essentially descriptions of persistent patterns in a system's phenomenology. However, this embodiment of the criterion of regularity is sufficiently broad that it can include regularities that are not immediately recognisable as visual patterns in a space-time diagram, e.g. the power-laws linking the size and frequency of certain extreme features in the Bak-Sneppen model (section 2.2, Figure 3).

Another embodiment of the criterion of regularity is the requirement of the existence of regular structures on several scales, which is often phrased as a requirement of the existence of a ***structural hierarchy*** (e.g. Ladyman et al., 2013, p. 12; p. 41). Ladyman et al. (2013, p. 41) describe the existence of hierarchical patterns in the phenomenology of a complex system in the following way:

> ... [A]n entity that is organised into a variety of levels of structure that interact with the level above and below and exhibit lawlike and causal regularities, and various kinds of symmetry, order and periodic behaviour.

This embodiment can therefore be regarded as more demanding than that of the existence of patterns or of macrolaws: it requires not only that such patterns and macrolaws are present in a system's behaviour but that they are present on several different scales. Furthermore, the relationships between some of these structures need to be regular, i.e. governed by macrolaws, as well.

The best known (and likely clearest) example of a model fulfilling this embodiment of the criterion is the Game of Life: started from a random initial distribution of values, the model's phenomenology usually contains some areas, in which its behaviour can best be described on the level of single grid cells, and some areas, in which patterns spanning three to five grid cells (e.g. gliders, boats etc) can be identified. Larger patterns involving more than 10 cells do not spontaneously arise but can be implemented into the CA through an intentional choice of its initial state (e.g. glider guns; space ships). Furthermore, macrolaws can also be formulated for the relationships between different patterns: e.g. gliders will periodically be created next to an (isolated) glider gun.

Requiring the existence of a structural hierarchy is a demanding embodiment of the criterion of regularity. Cellular automata, for example, which traditionally form a paradigmatic class of complex models spontaneously display only non-statical (and therefore potentially 'interacting') patterns on two scales: areas that are dominated by single-grid scale dynamics; and patterns at a larger scale involving three to five cells (e.g. the Game of Life). The appearance of robust larger structures usually requires the careful choosing of initial conditions. One-dimensional CAs appear to often display fluctuations on several scales. These fluctuations manifest as static, often triangular structures in spatial-temporal representations of the models' phenomenologies, which cam involve a large number of different spatial scales (e.g. the differently patterned areas of the CA110's phenomenology, Figure 2). However, since these are one-dimensional structures it is not obvious to me that the coexistence of these fluctuations should be described as 'interactions' in the same way that more complicated patterns can interact (e.g. gliders and other patterns in the Game of Life). In actual formulation (e.g. see quotation above), the number of different scales on which patterns need to be generated to constitute a hierarchy is often not specified. Accordingly, there seems to be no formal objection to counting systems which develop features on just two scales as fulfilling the criterion.

The terminology of 'macrolaws' and 'structural hierarchy' used to describe and label these embodiments of the criterion of regularity illustrates the use of pseudo-dynamical language and the construction of alternative dynamics. In particular, the notion of macrolaws seems to imply the construal of an alternative dynamics - consisting of these macrolaws - which could produce a phenomenology indistinguishable (in all important aspects) from the one that is currently produced by the actual (micro-)dynamics. While the use of such alternative dynamics constitutes a means of highlighting unexpected (with respect to its actual dynamics) aspects of its phenomenology (section 5), as discussed in section 3.2, an unreflective use of these terms harbours the danger of obfuscating the fact that these are descriptions and labels of phenomenological features.

3.2.2 Irregularity

There are several complexity definitions that require an embodiment of the criterion of irregularity (section 4). I use the term 'irregularity' as denoting 'an absence of regularity' (section 3.2.1). While irregularity is therefore a notion that conceptually contrasts with the criterion of regularity, the two criteria are not exclusive of each other. In section 4.1, I will show that some complexity definitions require both criteria to be fulfilled by a system's phenomenology to warrant the label 'complex' (e.g. by displaying regular and irregular behaviour simultaneously at different locations). Other complexity definitions disjunctively require both phenomenological criteria coupled with different dynamical criteria, i.e. by requiring that a complex model either has deterministic dynamics and displays irregular behaviour or has indeterministic dynamics and displays regular behaviour (section 4.3).

Embodiments of irregularity In contrast to the large number of technical embodiments of the criterion of aperiodicity used in definitions of chaos (e.g. Zuchowski, 2017; Werndl, 2009; Smith, 1998), definitions of complexity are often based on a simple visual notion of irregularity, i.e. requiring a ***disordered appearance*** (e.g. Wolfram, 2002). Technical embodiments of the criterion of irregularity are usually based on the notion of ***statistical entropy***. Reflecting the large number of different entropy definitions (e.g. Frigg and Werndl, 2011), there exist several technical realisations of such measures. Entropy measures based on the minimum length of description (of the phenomenology of a system), e.g. the Komologorov entropy (e.g. Chaitin, 1966, 1969), directly represent the notion that irregular regions of a system's phenomenology lack periodicity and are therefore not amenable to descriptions that make use of such regularities. However, entropy measures based on the length of description are usually uncomputable since they must be normalised by use of a universal language. In order to allow actual computability, other embodiments of the criterion are based on the assumption that high-phase space entropy, i.e. a uniform spread of the system's components' states through a suitably defined phase-space, is a suitable embodiment of the criterion of irregularity (Feldman and Crutchfield, 1998, e.g.). Recent investigations (e.g. Zuchowski, 2012) show that irregular phenomenologies are not the only ones to which such statistical entropy measures assign high values: for example, the highly-regular Sierpinski triangle created by the CA90 is also assigned high phase-space entropy values by many such measures.

Some complexity definitions include criteria that can be viewed as consequences rather than embodiments of irregularity: in particular, sensitive dependence on initial conditions (SDIC) has been named as a defining characteristic of complexity (e.g., in the list of definitions reviewed by Ladyman et al., 2013, p. 12). In chaos theory, where the criterion of aperiodicity is often more formally defined as an indistinguishability from the phenomenology of a Bernoulli process, SDIC is formally entailed by such embodiments of aperiodicity (e.g. Werndl, 2009). In complexity science, where irregularity is usually embodied more informally as an absence of regularity, SDIC also appears to be diagnosed informally as the development of very different phenomenologies for similar initial conditions. However, likely due to the lack of formal ties between the concepts, the use of SDIC as an independent criterion for complexity is relatively rare.

4 Analysis and comparison of complexity definitions

Having determined the criteria used to define complexity (section 3), I will now move up to the next layer tier of my analytic framework (section 1.1, Figure 4) and comparatively analyse several different complexity definitions: a determinism-based complexity definition by Wolfram (1984, 2002); an indeterminism-based complexity definition by Ladyman et al. (2013); and an inclusive complexity definition by Goldenfeld and Kadanoff (1999). I will show that each of these complexity definitions can be interpreted as requiring different combinations of different embodiments of the five criteria for complexity (section 3). Therefore, this section serves two purposes: (i) that of a comparative analysis of three complexity definitions, each of which can be viewed as an example of a class of similar definitions; and (ii) that of a demonstration of the merits of the tiered framework for the analysis of definitions (section 1.1) by giving a clear exposition of the structure of complexity definitions.

In section 3.2, I discussed the fact that, in complexity science, distinctions between the dynamical and phenomenological features of a system are often not strictly upheld and, in some cases, is blurred by the use of pseudo-dynamical terminology to describe the latter. Accordingly, identifying the combination of criteria used by a specific definition can require some interpretative work. Furthermore, not all definitions explicitly identify all dynamical criteria they seem to be requiring. In these cases, I will reconstruct the dynamical criteria required from the catalogue of models studied, e.g. if the definition of complexity is taken to apply to deterministic models only, then this will be counted as requiring the criterion of determinism. As discussed in section 1, my analysis here is therefore a work of rational reconstruction: I do not claim that it provides the sole possible approach to the analysis of complexity definitions but hope that its merits will be amply demonstrated in the clarity with which their structure is revealed and in its use to derive a minimal definition of complexity (section 5).

4.1 Wolfram's determinism-based complexity definition

Wolfram (1984, 2002) provides an indirect definition of complexity through his classification system for the behaviour of CAs. This system was initial developed to classify the behaviour of one-dimensional, binary, nearest-neighbour CAs (e.g. Wolfram, 1984). However, it is also used in Wolfram (2002), which investigates a very large number of different, deterministic CAs and is intended as a comprehensive study of complex models (Wolfram, 2002, Chapters 1-3). Accordingly, the classification system, and the implicit definition of complexity it contains, can be taken to apply generally.

Wolfram (2002, p. 231) describes the four classes of behaviour observed in CA-like systems in the following way:

> In class 1, the behaviour is very simple and almost all initial conditions lead to exactly the same uniform final state.
> In class 2, there are many different possible final states, but all of them consist just of a certain set of simple structures that either remain the same forever or repeat every few steps.
> In class 3, the behaviour is more complicated, and seems in many respects random, although triangles and other small-scale structures are essentially always at some level seen.

> ... [C]lass 4 involves a mixture of order and randomness; localised structures are produced which on their own are fairly simple, but these structures move around and interact with each other in very complicated ways.

Subsequently, class 4 is identified as the class of models that should be labelled 'complex' (p. 230): therefore, the description associated with this class implicitly provides a definition of complexity.

In the remainder of this section, I will show that this definition can be viewed as requiring three of the five criteria for complexity identified in section 3: determinism (section 3.1.2); regularity (section 3.2.1); and irregularity (section 3.2.2).

Determinism Wolfram (2002, 1984) does not explicitly require determinism as a criterion. However, the fact that the classification scheme, and therefore the definition of complexity, is intended to apply only to deterministic models and systems can be deduced from (i) the choice of models discussed in Wolfram (2002); and (ii) his repeated description of the rule sets of these models as 'simple' (e.g. p. 51).

With respect to aspect (i), it is important to keep in mind that Wolfram (2002) is intended as a comprehensive review of those systems whose investigation, according to Wolfram, will constitute 'a new kind of science'. Accordingly, the fact that all of the models discussed in Wolfram (2002, Chapters 2-3) are deterministic is a strong indication that complexity is seen as an exclusive feature of such models, i.e. that determinism is indirectly required as a criterion.

With respect to aspect (ii), Wolfram (2002, e.g. p. 51) repeatedly describes these systems and models as having 'simple' dynamics. Thereby, 'simplicity' implies that the rule set of a given system can be graphically represented as a transition diagram: namely, a diagram that assigns to each state of a cell and of those in its neighbourhood unique states the cells will transition to during the next time step. Assigning a unique state to each configuration is equivalent to a deterministic description, of course (section 3.1.2). Consequently, I take Wolfram's (2002) requirement of 'simple' dynamics to constitute an embodiment of the criterion of determinism. Therefore, I maintain that Wolfram (2002, 1984) definition of complexity requires the criterion of determinism, in the embodiment of the requirement of underlying processes that can be represented as uniquely defined transition maps.

Since most of the models in Wolfram (2002) are many-component models, it could be argued that this criterion (section 3.1.1) should also be viewed as being implicitly required. However, I maintain that not including this criterion leads to a more truthful reconstruction of the definition of complexity used by Wolfram (2002) for two reasons: (i) Wolfram's catalogue of relevant systems includes models with very few components; (ii) in contrast to the simplicity of the rule sets, the property of having many components is not highlighted as an important feature of these systems. Accordingly, I assume that Wolfram's definition of complexity does not require the criterion of many components. However, it should be noted that this decision does not crucially influence any of the arguments made in this paper.

Regularity and irregularity Since the part of the definition of complexity implied by Wolfram's classification scheme is a phenomenological one, its requirement of the criteria of regularity and irregularity are directly apparent in the quotation displayed above. The required embodiments of these criteria are the presence of visually recognisable patterns and the absence of such patterns, respectively. Wolfram has been adamant that the presence or absence of patterns can easily be detected visually (e.g., while being interviewed by Gershenson, 2008).

An important aspect of this definition is that these two - *prima facie* incompatible - phenomenological criteria are required simultaneously. As apparent in the definition as quoted above, this can be accomplished by requiring the existence of patterns, i.e. periodicity, in some spatio-temporal regions of the system's phenomenology and the absence of patterns, i.e. aperiodicity, in others. In other words, this definition of complexity requires deterministic dynamics that lead to both periodic as well as aperiodic behaviour.

It is immediately obvious that the CA110 (section 2.1, Figure 2) can be viewed as a paradigmatic example of Wolfram's complexity definition. However, in our set of case studies, the CA110 is the only model that would be labelled 'complex' under this definition: the Bak-Sneppen model (section 2.2) does not fulfil the criterion of determinism and is therefore excluded; and the logistic equation's phenomenology (section 2.3, Figure 4) does not simultaneously display regular and irregular behaviour (it shows class 3 instead of class 4 behaviour according to Wolfram's classification scheme) and is therefore also excluded.

4.2 Ladyman's et al. indeterminism-based complexity definition

Ladyman et al. (2013, p. 57) propose a complexity definition derived from a survey of existing complexity definition and a subsequent attempt to distil the most important features of those definitions:

> A complex system is an ensemble of many elements which are interacting in a disordered way, resulting in robust organisation and memory.

In the following, I will show that this definition of complexity requires three of the five criteria for complexity identified in section 3: the existence of many components (section 3.1.1); indeterminism (section 3.1.3) and regularity (section 3.2.1). In another part of their paper, Ladyman et al. (2013, Section 3) also discuss which technical embodiments the latter criterion should assume, namely the requirement of a structural hierarchy.

Existence of many elements It is immediately apparent from the quotation that this criterion is part of the definition. Ladyman et al. (2013, pp. 57) further specifies that these elements should be "similar in nature" and describe a number of examples of such systems: gases with different molecules; bodies composed of cells; flocks of animals.

Indeterminism While Wolfram (2002) uses the term 'disordered' to describe the phenomenology of complex systems, Ladyman et al. (2013, p. 58) use it as a synonym of indeterminism, i.e. to describe dynamics that do not have unique outcomes for given initial values and that therefore contain probabilistic elements.[1] Their catalogue of examples of complex systems implies that such dynamics do not need to be random, i.e. equiprobable, but must contain some indeterministic elements. Ladyman et al. (2013, p. 58) also emphasize that fully deterministic systems are disqualified from being labelled complex:

[1] This impression was confirmed in personal conversation with Karoline Wiesner during the *Physics in Society* workshop at the LMU, Munich, 22-23/7/2016.

> Disorder is a necessary condition for complexity simply because complex systems are precisely those whose order emerges from disorder rather than being built into them.

The fact that the definition describes the phenomenological criterion as 'resulting' from the two dynamical ones, indicates that Ladyman et al. (2013) assume that these two criteria will also be part of the sufficient conditions for the occurrence of complexity. (Although, given that not all indeterministic, many-components systems fulfil the criterion of regularity, these two conditions by themselves cannot constitute the full set of sufficient conditions for the occurrence of complexity.)

Regularity As discussed in section 3.2.1, in complexity science, the terms 'memory' and 'organisation' are often used as synonyms for temporal and spatial regularity, respectively. Furthermore, Ladyman et al. (2013, p. 60) make it clear that they view robust organisation as being hierarchical, i.e. as pattern formation on several scale-levels. The embodiment of regularity proposed here is therefore that of the existence of a structural hierarchy in a system's phenomenology. In an earlier part of their study, Ladyman et al. (2013, section 3) also discuss the possibility that a requirement of medium phase-space entropy values could be used as a technical embodiment of the criterion of periodicity. However, their explicit definition does not directly refer to this embodiment and recent conversations with the authors (cc. footnote 1) indicate that such entropy measures are no longer seen as suitable formalisations of this criterion.

It should be noted that only local regularity is required by Ladyman et al. (2013), i.e. (hierarchical) pattern formation must be evident in a complex system's phenomenology but it is not required that the system's behaviour is regular everywhere. Accordingly, systems with phenomenologies that are locally regular and locally irregular can also fulfil the indeterminism-based complexity definition. However, local irregularity is not required: systems that are fully regular but fulfil the dynamical criteria outlined above would therefore also judged to be complex.

From this analysis, it is immediately obvious that the complexity definition by Ladyman et al. (2013) differs significantly from the determinism-based complexity definition by Wolfram (2002). Besides explicitly requiring the criterion of the existence of many elements, their definition requires a combination of indeterministic dynamics and regular behaviour. In contrast, Wolfram (2002) requires a combination of deterministic dynamics with behaviour that is both regular and irregular. Since the dynamical criteria of determinism and indeterminism are exclusive of one another, the two complexity definitions are likewise exclusive, i.e. their extensions do not overlap and no system will be classified as being complex by both definitions.

The exclusivity of the two definitions can be illustrated through the classification of our case studies: the CA110 (section 2.1), which is complex according to Wolfram's definition (section 4.1), does not fulfil the criterion of indeterminism and is therefore not complex according to Ladyman's et al. complexity definition. Similarly, the Bak-Sneppen model (section 2.2), which could be seen as a paradigmatic case for Ladyman's et al. indeterministic definition, is not complex according to Wolfram's deterministic definition (section 4.1). The third model in our set of case studies, the logistic equation (section 2.3), is excluded from the extension of the indeterministic complexity definition by failing to meet two of its criteria:

it is neither deterministic nor does its phenomenology show any kind of regularity.

4.3 Goldenfeld & Kadanoff's inclusive complexity definition

The definition of complexity given by Goldenfeld and Kadanoff (1999, p. 87) is very simple:

> To us, complexity means that we have structure with variation.

Of course, this definition by itself is too short to be analysable. However, the accompanying article discusses several examples of systems that possess "structure with variation" from which a tighter notion of the content of this definition can be constructed:

- Chaotic systems: Systems which obey "simple" (e.g. p. 87) – deterministic – laws, i.e. have structured dynamics, and have irregular phenomenologies, i.e. have variable phenomenologies.

- Spin-glass type systems: Systems with many components that obey simple, deterministic laws, i.e. have structured dynamics, and have phenomenologies that are both regular and irregular, i.e. have some element of variation in their phenomenology.

- Fluid systems with diffusion: Systems with many component that obey indeterministic laws, i.e. have varied dynamics, and have phenomenologies that are both regular and irregular, i.e. have some element of structure in their phenomenology.

Accordingly, the definition by Goldenfeld and Kadanoff (1999) seems to roughly equate 'structure' with either the criterion of determinism or of regularity and to equate 'variation' with either the criterion of indeterminism or of irregularity. Under this interpretation, only systems that have structured dynamics and varied behaviour, i.e. that have deterministic dynamics and (at least locally) aperiodic behaviour, or that have varied dynamics and structured behaviour, i.e. that have indeterministic dynamics and (at least locally) periodic behaviour should be included in the definition's extension. This definition is therefore based on four of the five criteria for complexity identified in section 3: determinism (section 3.1.2); indeterminism (section 3.1.3); regularity (section 3.2.1) and irregularity (section 3.2.2). However, the definition only requires the fulfilment of one of two distinct combinations of these criteria: either of (i) determinism coupled with irregularity; or of (ii) indeterminism coupled with regularity.

Goldenfeld and Kadanoff (1999, pp. 88-89) do not specify any specific embodiments for these criteria. Instead, their discussion indicates that the choice of precise embodiment for each criterion should be system-specific: e.g. typical measures of stochasticity as embodiments of irregularity in chaotic systems; visual identification of irregularity in spin-type systems; statistical power-laws as embodiments of regularity in fluid systems with diffusion.

Goldenfeld & Kadanoff's (1999) definition is the most inclusive of the three definitions discussed here. In particular, its extension contains chaotic systems, which fulfil combination (i) of the required criteria. All systems that are diagnosed as complex according to the determinism-based complexity definition (section 4.1) will also fulfil this combination of criteria and will therefore also be included. Likewise, all systems that are diagnosed as complex according to the indeterminism-based complexity definition (section 4.2) fulfil combination (ii) of the criteria for complexity and are therefore also included.

Table 1: Combinations of criteria required by three prevalent complexity definitions (section 4). For the inclusive definition, the two possible combinations are superscribed (i) and (ii), as defined in section 4.3.

	Deterministic definition	**Indeterministic definition**	**Inclusive definition**
Many components		X	
Determinism	X		$X^{(i)}$
Indeterminism		X	$X^{(ii)}$
Regularity	X	X	$X^{(ii)}$
Irregularity	X		$X^{(i)}$

The inclusivity of Goldenfeld & Kadanoff's definition can be illustrated through the classification of our three case studies (section 2): all three models will be classified as complex according to this definition. In particular, the CA110 (section 2.1) and the logistic equation (section 2.3) both fulfil combination (i) of the required criteria, while the Bak-Sneppen model (section 2.2) fulfils combination (ii).

5 A minimal definition of complexity

In this section, I will use the results of my analysis of the three complexity definitions (section 4) to show that all three definitions require contrasting dynamical and phenomenological criteria: namely, they require either combinations of criteria that pair determinism with irregularity or indeterminism with regularity. From this, I will derive a minimal definition of complexity, i.e. a description of the concept that underlies all of these definitions (section 1.1).

In section 5.2, I will argue that this contrast between the dynamics and the phenomenology of a system, i.e. the concept behind all complexity definitions, can be interpreted as a type of epistemological emergence. I will therefore argue that it is possible to (minimally) define complexity as a type of emergence, but that the kind of emergence associated with complex systems is a relatively weak one.

5.1 Dynamics-phenomenology contrasts as the conceptual heart of complexity science

As discussed in section 4, each of the three analysed complexity definitions requires different combinations of the five criteria for complexity. These combination are displayed comparatively in Table 1. The table illustrates some of the results already discussed in section 4: the determinism-based and indeterminism-based definition, which require exclusive dynamical criteria, have non-overlapping extension; the extension of the inclusive definition includes the extensions of the deterministic and indeterministic definition. In addition, the inclusive definition is the only definition whose extension includes chaotic systems. These systems will be excluded from the indeterminism-based definition through the requirement of the dynamical criteria of the existence of many components and of indeterminism. Chaotic systems will generally also be excluded from the determinism-based complexity definition through the requirement of two phenomenological criteria to occur simultaneously: while chaotic systems may have periodic and aperiodic phases parameter regimes, these properties are not displayed simultaneously in the phenomenology of a model.

Since the determinism-based and indeterminism-based complexity definition have non-overlapping extension, one can easily find systems that will be diagnosed as being complex according to one definition but not according to the other. For example, the CA110 (section 2.1) is complex according to Wolfram's determinism-based definition but not according to Ladyman's et al. indeterminism-based definition (section 4.2); the Bak-Sneppen model (section 2.2) is complex according to the indeterminism-based definition but not according to the determinism-based one. *Prima facie* it might therefore appear impossible to identify a unifying concept behind these different definitions. Attempts to do so have usually relied on a metaphorical approach: the concept of complexity is supposed to be captured by a unifying slogan. In the following, I will review the two most prominent metaphorical descriptions of the concept of complexity and will show that they are best interpreted as restatements of particular complexity definitions. As such, I think that this approach to finding a minimal definition of complexity is not a promising one. Rather, these metaphorical definitions should be interpreted as highlighting aspects of each definition that can be used to define the concept of complexity in a more technically throughout way.

Interpretation of metaphorical descriptions of the concept of complexity The analysis and comparison of different complexity definitions (section 4) can help to make more precise some notorious metaphorical descriptions of the concept of complexity: namely, the notions of *complexity being located between chaos and order* and of *self-organisation*. Each of these terms has been criticised for being overly metaphorical and for not being applicable to a number of well-known complex models (e.g. Frigg, 2003; Zuchowski, 2012). However, viewed in the context of the coexistence of two exclusive complexity definitions, one can assign clearer meanings to these terms. Namely: the notion of complexity as being conceptually located between chaos and order appears to capture the simultaneous requirement of the two phenomenological criteria of regularity and irregularity in combination with the requirement of the dynamical criterion of determinism in the determinism-based definition; the notion of self-organisation appears to describe the requirement of the phenomenological criterion of periodicity in combination with the dynamical criterion of indeterminism in the indeterminism-based definition.

The determinism-based definition of complexity requires the phenomenology of complex systems to display both irregular and regular features; due to the fact that the criterion of determinism is also required, the irregular parts of this phenomenology could be labelled chaotic in the sense of some chaos definitions (e.g. Smith, 1998; Zuchowski, 2017). If it is taken to describe the coexistence of regular and irregular dynamics in deterministic (albeit possibly many-component) systems, then the metaphor of complexity being located between chaos and order acquires a sharper conceptual meaning. However, the making more precise of this concept also entails a new exclusivity. In particular, none of the models in the extension of the indeterminism-based definition of complexity fall under this description since (by definition) they lack the deterministic dynamics necessary for the diagnosis of chaos. Of those models to which the inclusive definition of complexity applies, only those that fulfil combination (i) of the required criteria, i.e. which fulfil the dynamical criterion of determinism and the phenomenological criterion of irregularity, and that additional show regular phenomenological features (which are not required by the inclusive definition), could be described as being located 'between order and chaos'. Therefore, under this interpretation, the metaphor becomes a rephrasing of one particular complexity definition – the determinism-based one – only. In the set of our three case studies (section 2), only the CA110 would be covered by the metaphor.

The indeterminism-based definition of complexity requires indeterministic dynamics in combination with phenomenologies that show regular features. In section 3.2.1, I discussed the fact that the term 'organisation' is often used as paraphrasing of regularity, in particular, in relation to the embodiment of the criterion as the existence of a structural hierarchy. If the term 'self-organisation' is taken to describe precisely this combination of features, i.e. the existence of indeterministic dynamics and regular phenomenological features, then its conceptual content is well-defined. However, this new conceptual precision again comes with a new level exclusivity: none of the models in the extension of the determinism-based complexity definition, which lack the required indeterministic dynamics, could then be called self-organised. Furthermore, of those models that are complex according to the inclusive definition, only models that fulfil combination (ii) of the required criteria, i.e. those that fulfil the dynamical criterion of indeterminism and the phenomenological criterion of regularity, should be described as self-organised. In the set of our three case studies (section 2), only the Bak-Sneppen model would be covered by the metaphor.

Arguably the most surprising result of this analysis is the fact that the concepts of 'being located between order and chaos' and of 'self-organisation' – if their conceptual content is sharpened to move them beyond the mere metaphorical level – are exclusive of each other. That is: since these concepts appear to be encapsulations of the particular combinations of criteria required by two different definitions of complexity, whose extensions do not overlap, no system can be both located between order and chaos as well as be self-organised. I therefore maintain that they are not suitable to serve as descriptions of the concept of complexity, which (according to analytic framework underlying my analysis, section 1.1, should ideally be reflected in all associated definitions. However, I will now argue that these metaphors highlight a shared feature of these definitions that can be used for a minimal definition of complexity.

Dynamics-phenomenology contrasts as the conceptual heart of complexity As outlined by Zuchowski (2017, Chapter 3), the co-existence of several, partly exclusive definitions and concepts in a scientific field is not necessarily a sign of nefarious conceptual divides. Rather, it can be interpreted as a useful means of highlighting the particular features of a particular class of systems that are most interesting to researchers in that field. If the two concepts of 'being between order and chaos' and 'self-organisation' are interpreted as such means of highlighting interesting features, a feature characteristic of both concepts becomes apparent: they both highlight a contrast between the dynamics of a system and its phenomenology.

Underlying the assumption that indeterministic dynamics contrast with regular phenomenological features and that deterministic dynamics contrast with irregular phenomenological features, is - of course - a baseline expectation that deterministic dynamics lead to regular behaviour and that indeterministic dynamics lead to irregular behaviour. That this expectation is (or used to be until the advent of complexity science) mainstream among scientists is frequently stressed by complexity scientists (and by chaos scientists, e.g. May, 1976, p. 459). For example, Wolfram (2002, pp. 39-40) writes:

> Yet at first this [the behaviour of (deterministically) complex CAs] may seem almost impossible to believe. For it goes against some of our most basic intuitions about the way things normally work. For our everyday experience leads us to believe that an object that looks complicated must have been constructed in a complicated way.

All of the complexity definitions discussed in section 4, and the two prominent metaphorical descriptions analysed here, highlight this precise contrast between existing intuitions about the phenomenologies of systems with particular dynamics and their actual phenomenologies. Using the terminology that I have introduced here, all of them therefore formalise a specific version of this contrast between the dynamics and the phenomenologies of the specific class of systems in their respective extensions. Accordingly, I maintain that this contrast between the dynamics of a model and its phenomenology, judged against mainstream intuitions about the relationship between these two parts of a model, appears to constitute the conceptual heart of complexity science. It can therefore serve as a minimal definition of the concept of complexity: complexity describes a contrast between the degree of determinism of the dynamics and the regularity/irregularity of the phenomenology of a system. Interpreted in this light, the different complexity definitions, and the metaphorical descriptions associated with them, can be viewed as means to identify and to highlight such contrasts in different classes of systems, i.e. as means of making this general concept applicable to specific classes of systems.

5.2 Dynamics-phenomenology contrasts as epistemological emergence

In section 5.1, I identified the existence of a contrast between the (expectations about) the dynamics of a system and its phenomenology as the core concept behind the label 'complex'. It is easy to see that this contrast can also be interpreted as a kind of epistemological emergence (section 5.2). As a matter of fact, the quotation by Hempel and Oppenheim (1948, p. 147) displayed above, describes emergence as precisely the disappointment of reasonable expectations about the behaviour of a system based on knowledge about the system's components. In my terminology, this definition of emergence maps onto the requirement of a contrast between the expectations formed from the known dynamics of a system and its actual phenomenology. Given that this contrast is grounded in the intuitions that scientists have formed from their previous work with models and systems, i.e. the expectation that deterministic dynamics lead to regular phenomenologies and that indeterministic dynamics lead to irregular phenomenologies, this notion of emergence is clearly an epistemological one.

The fact that the core concept behind different definitions of complexity can be viewed as a kind of emergence explains why various versions of this latter notion have been associated with complexity (e.g. Holland, 1998; Kauffman, 1995; Davies, 2003; Kauffman, 2008). The different complexity definitions surveyed in section 4 can be interpreted as articulations of this kind of emergence for different classes of models. As a kind of epistemological emergence, 'complex emergence' appears to be unique in its focus on irregular behaviour emerging from deterministic dynamics or regular behaviour emerging from indeterministic dynamics.

The fact that the type of emergence that is associated with the concept of complexity is epistemological in nature does not mean that it is 'trivial' or 'uninteresting'. While my analysis doe not bear out inflationary claims about the importance of complex emergence in nature (e.g. Morowitz, 2002; Gregersen, 2003), it does support the view that complexity is a concept used to highlight a class of systems that needs new investigative approaches (e.g., as advocated by Wolfram, 2002; Kauffman, 2008). In particular, new technical tools are needed to extrapolated from the (observed) phenomenology of these systems to their underlying dynamics; the case of chaos theory – which under this minimal definition would indeed be an exploration of the same concept in a particular manifestation, namely for systems with few components - shows that the development of such techniques is both fruitful as well as technically demanding (e.g. Zuchowski, 2017, Chapter 4).

6 Conclusion

In this paper, I analysed the use of phenomenological and dynamical criteria in different complexity definitions (sections 3 and 4) and used this analysis to provide a minimal definition for the concept underlying all of these definition (section 5.1). Thereby, I used a tiered analytical framework (section 1.1, Figure 1) that distinguishes between a general concept, which can be defined through a minimal definition; different definitions associated with this concept; criteria that are used in these definitions; and technical embodiments of these criteria. My results were illustrated on three well-known models in complexity science (section 2): the CA110 (section 2.1); the Bak-Sneppen model (section 2.2); and the logistic equation (section 2.3).

In section 3, I identified five core criteria for the diagnosis of complexity: three dynamical criteria (section 3.1) and three phenomenological criteria (section 3.2). The three dynamical criteria are the existence of many components (section 3.1.1); determinism (section 3.1.2); and indeterminism (section 3.1.3). The two latter criteria thereby require the dynamics of a system to either allow only one unique development from a given initial value or to allow different developments from a given initial value, i.e. to contain probabilistic elements, respectively. The two phenomenological criteria for complexity are regularity (section 3.2.1) and irregularity (section 3.2.2). Embodiments of regularity are usually measures of the number of patterns identifiable in a system's behaviour while embodiments of irregularity can either be measures of the absence of regularity or measures that encapsulate similarities of the system's phenomenology with that of a truly stochastic system.

I then analysed the use of these criteria in three different complexity definitions: Wolfram's determinism-based complexity definition (section 4.1); Ladyman's et al. indeterminism based complexity definition (section 4.2); and Goldenfeld & Kadanoff's inclusive complexity definition (section 4.3). Each of these definitions can be seen as an example of a class of similar definitions. I showed that each definition can be interpreted as requiring different combinations of different embodiments of the five criteria for complexity (Table 1). In particular, the determinism-based definition requires the dynamical criterion of determinism and the simultaneous fulfilment of the phenomenological criteria of regularity and irregularity. The indeterminism-based definition requires the dynamical criteria of the existence of many components and of indeterminism. Due to the fact that they require dynamical criteria that are exclusive of another, the determinism- and indeterminism-based definitions are mutually exclusive, i.e. systems that fulfil the criteria of one of these definitions cannot fulfil the criteria of the other one. Among the three case studies considered, only the CA110 fulfilled the criteria of the determinism-based definition and only the Bak-Sneppen model fulfilled those of the indeterminism-based complexity definition. Both definitions exclude chaotic systems from also being labelled complex: the determinism-based definition does so by requiring the phenomenological criterion of regularity while the indeterminism-based definition does so by requiring the dynamical criterion of indeterminism. The logistic equation is therefore not labelled as complex by either the determinism- or the indeterminism-based complexity definition.

The third complexity definition analysed here, the inclusive complexity definition, possesses an extension that includes the extensions of the determinism- and indeterminism based definition. It also includes chaotic systems. This is achieved by requiring the fulfilment of one of two possible combinations of criteria: either the dynamical criterion of determinism coupled with the phenomenological criterion of irregularity; or the dynamical criterion of indeterminism coupled with the phenomenological criterion of regularity. The disjunctive nature of this definition makes it applicable to a wide range of systems. In particular, all three of the case studies are complex according to this definition.

Finally, I used the results from my analysis of different complexity definitions to shed light on the general concept of complexity and to provide a minimal definition of this concept. In section 5.1, I showed that the three complexity definitions discussed all share one feature: they all require the existence of a contrast between the classified systems' dynamics and phenomenology. This is achieved by requiring combinations of the dynamical and phenomenological criteria that defy mainstream expectations about how these criteria should be related, i.e. by pairing the criterion of determinism with that of irregularity and that of indeterminism with that of regularity. I also showed that two prevalent metaphorical descriptions in complexity science, 'being between order and chaos' and 'self-organisation', could be mode more precise if interpreted as descriptions of two such dynamics-phenomenology contrasts, namely the ones associated with the determinism- and indeterminism-based definitions, respectively. However, as they conceptualise only specific dynamics-phenomenology contrasts, these metaphorical descriptions are not general enough to serve as minimal definitions of complexity. Instead, I proposed the following minimal definition for complexity: a contrast between the degree of determinism of the dynamics of a system and the regularity/irregularity of its phenomenology.

In section 5.2, I argued that the dynamics-phenomenology contrast that underlies all complexity definitions and defines the concept of complexity constitutes a kind of epistemological emergence. It would therefore be possible to use this framework to define 'complex emergence' as the emergence of phenomenological features that defy our expectations (derived from knowledge about a given system's dynamics) about the phenomenology of this system. In particular, complex emergence describes the emergence of irregular features in the behaviour of deterministic systems and the emergence of regular features in the behaviour of indeterministic systems.

The fact that these particular combinations of dynamical and phenomenological features are perceived as contrasting or emergent might raise questions about the anthropocentric nature of the concept of complexity, which appears to be deeply entwined with our expectations about which dynamics should lead to which behaviour. However, such questions are raised by the very notion of epistemological emergence and their discussion exceeds the scope of this paper.

I hope that my analysis has clearly shown that these dynamics-phenomenology contrasts form the conceptual heart of complexity science. Under this interpretation, the determinism- and indeterminism-based definition are viewed as different articulations of this core concept for different classes of models. Accordingly, I view these two definitions not as rivals for the one true definition of complexity but as complementary expressions of one single concepts. By allowing the articulation of different but, equally relevant, kinds of dynamics- and phenomenology contrasts, their coexistence appears to be epistemically useful rather than divisive. In this sense, complexity science appears to be similar to other fields of science (e.g. as discussed in section 1.1, chaos theory; the study of social interactions; climate science) in which the need to combine different models, techniques and methods renders the coexistence of several definitions of the field's core concepts expedient.

I also suggested that dynamics-phenomenology contrasts should be seen as the defining feature of complexity science; a feature that is reflected in different form in all complexity definitions and in the most prominent metaphorical descriptors of the field. In this context, it is useful to question whether inclusive definitions, like the one discussed in section 4.3, might not be too inclusive. In particular, I think there is some merit in avoiding the inclusion of chaotic systems into the extensions of complexity definitions. My argument for this recommendation is not based on any conceptual qualms with this definition, which is a valid articulation of the concept of complexity as minimally defined above, but on a methodological worry. While some definitions of chaos require deterministic dynamics and irregular phenomenologies, there are also chaos definitions which are not articulations of such a dynamics-phenomenology contrast (e.g. Zuchowski, 2017; Smith, 1998). Furthermore, the methodology of chaos theory is distinct from that of the investigation associated with models covered by the determinism- and indeterminism-based complexity definition: the methods used to investigate this one particular kind of contrast and other kinds of chaos appear to be of little direct use for the investigation of the systems targeted by the determinism- or indeterminism based definition. Accordingly, a desire for maximum conceptual clarity and epistemic usefulness should dictate that any complexity definition avoids the unnecessary inclusion of already categorised systems and focuses on identifying systems that feature novel types of dynamics-phenomenology contrasts, e.g. as done by the determinism- and indeterminism-based definition. As discussed in section 3.1.1, an easy way off excluding the majority of chaotic systems, including the logistic equation used as a case study in this paper, is to universally require the criterion of the existence of many components. Accordingly, a somewhat stricter minimal definition for complexity might be entertained, which defines complexity as the existence of dynamics-phenomenology contrasts in systems with many elements. As argued above, I think that such a stricter minimal definition has methodological advantages but would still maintain that conceptually the most important defining feature of complexity are dynamics-phenomenology contrasts.

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